

Current-potential characteristics for a two-electrode device with constant pressure.

predictions. Accordingly, most of the information concerning the various forms of gaseous conduction is empirical. In the present description, it will be possible to mention the main features of a few of these modes.

The **illustration** shows a sample voltage-current characteristic for a two-electrode device with constant pressure. It is assumed that there is a constant source of ionization which could be any of the primary sources previously discussed. In region *A*, the current first rises and then over a limited range is relatively constant as the voltage across the electrodes is increased. The initial rise is the result of the collection of charges which were either recombining or diffusing to the walls. The nearly constant current region is the result of the collection of almost all of the charges.

In region *B*, further increase in voltage produces an increase in current. Here, ionization by electron impact is occurring. The situation is described by specifying that each free electron makes α additional ion pairs in traveling 1 cm in the direction of the field. The number of ion pairs produced per second in 1 cm at a distance x from the cathode (assuming parallel plate electrodes) is given by Eq. (3),

$$n = n_0 e^{\alpha x} \quad (3)$$

where n_0 is a constant depending on the initial number of electrons. This is a form of the Townsend equation, and α is the first Townsend coefficient. In the region *B* in the illustration, the increase in current represents an increase in α . Near the end of this region, the current increases more rapidly with applied field. Here, additional effects are taking place, such as the photoelectric process and secondary emission. This situation is described by

Eq. (4), where β is the second Townsend coefficient,

$$i = i_0 \frac{(\alpha - \beta) e^{(\alpha - \beta)x}}{\alpha - \beta e^{(\alpha - \beta)x}} \quad (4)$$

cient, i_0 is the initial electron current at the cathode, and i is the anode current as a function of plate separation x ; β is also a function of electric field.

At the end of the region, the slope becomes infinite, and if the external resistance is not too large, the current will jump in a discontinuous fashion. The transition is referred to as a spark, and the potential at which it occurs is the breakdown or sparking potential. The region *B* is called a Townsend discharge and is not self-sustained. Thus, if the source of primary ionization were removed, the discharge would cease. See BREAKDOWN POTENTIAL; ELECTRIC SPARK.

As the potential reaches the sparking potential, a transition occurs to the region *C*. This is the self-sustained glow discharge region. Over an extensive current range, the voltage drop remains substantially constant. During the current increase, a glow occurs at the cathode, and at the upper end of the range, the cathode is completely covered. At this point, a further current increase can be achieved only if the potential drop across the discharge is increased. This portion of the characteristic is known as the abnormal glow. Throughout this portion of the discharge characteristic curve, secondary effects are quite important. Particularly vital are the effects of cumulative ionization and secondary emission at the cathode. See GLOW DISCHARGE.

Further increase in current leads to another mode of discharge, the arc. This is shown as region *D* in the illustration. Characteristic of this mode is the low cathode potential fall and the very high current density. It is generally felt that the predominant effect in the production of the large number of electrons at the cathode necessary for the arc is thermionic emission. This is consistent with the very high temperatures known to exist either generally or locally on the cathode. Although the arc type of discharge has very great commercial value, the mechanism of its operation is not very well understood.

In addition to these general types of conduction, there are very special cases of considerable interest. Some of these are the corona discharge, radiofrequency or electrodeless discharge, hot-cathode discharge, and discharges in the presence of a magnetic field.

Glenn H. Miller

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Electrical conductivity of metals

The property of a metal that measures its ability to conduct electricity, following Ohm's law. Electrical conductivity σ (measured in $\Omega^{-1} \cdot \text{m}^{-1}$) is the

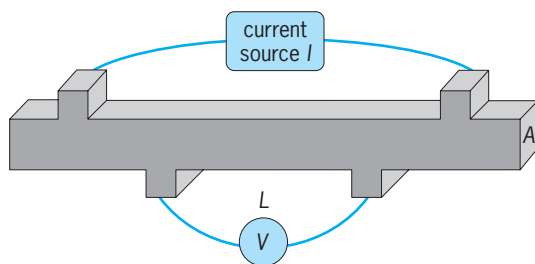


Fig. 1. Schematic of a four-probe measurement of electrical conductivity. Current I is fed through the outer leads, and voltage drop V is measured on the inner leads. In this way the contact potential drop experienced by the applied current is localized at the junctions with the outer leads. Measuring V with minimal current through the voltmeter minimizes the contact potential contribution to the measured conductance $G = I/V$.

reciprocal of the resistivity ρ [in $\Omega \cdot \text{m}$, where resistance R in Ohms (Ω) is voltage drop V in volts (V) divided by current I in amperes (A)]. Ohm's law, $V = IR$, may also be written in the form of Eq. (1), where j is current density (current per

$$j = \sigma E \quad \text{or} \quad E = \rho j \quad (1)$$

unit area, I/A , measured in A/m^2) and $E = V/L$ is electric field or electrical potential gradient, measured in V/m , where V is the voltage drop measured across a length L of material (Fig. 1). The conductivity σ is an intrinsic property of a pure material, related to the measured conductance $G = I/V$ via $G = \sigma A/L$, just as resistance $R = V/I$ relates to resistivity ρ via $R = \rho L/A$. Thus a large-gauge conductor [large area (A)] of short length has a high conductance, but the physical dimensions do not affect the conductivity σ or the resistivity ρ . Positive current flows from higher to lower voltage, and σ is never negative. See ELECTRICAL RESISTANCE; ELECTRICAL RESISTIVITY; OHM'S LAW.

Metals versus insulators. Metals have large conductivity (typically greater than $10^5 \Omega^{-1} \cdot \text{m}^{-1}$ but never greater at room temperature than that of silver, $0.66 \times 10^8 \Omega^{-1} \cdot \text{m}^{-1}$.) As temperature T is lowered, conductivity almost always increases in metals (Fig. 2). These properties contrast with semiconductors and insulators, which have smaller conductivity and different temperature dependence. The reason concerns the spectrum of quantum excited states in matter (Fig. 3). Outer electrons (occupying frontier orbitals in chemical language) have a continuous range of energy levels in solids, called a band of quantum states. In metals the highest-energy occupied state and the least-energy empty state differ only infinitesimally in energy, whereas in semiconductors or insulators they are separated by a significant energy gap. In both semiconductors and metals the states that correspond to the frontier orbitals are mobile, meaning that they permit electrons to travel from atom to atom. In semiconductors the energy gap is moderate (typically 0.5–3.0 eV). If electrons are introduced into empty states above the gap, a semiconductor behaves much like a metal. In an insulator the energy gap is larger, 5 eV or more, and elec-

trons typically get trapped on atoms or defects, rendering them comparatively immobile. See BAND THEORY OF SOLIDS; ELECTRIC INSULATOR; HOLE STATES IN SOLIDS; SEMICONDUCTOR.

Propagation of electrical current. To outside observers, electrical current flow in a wire resembles water flow in a pipe, but the analogy does not explain what is going on internally. In water flow, molecules have a net drift velocity, which is preserved by intermolecular collisions. Locally there is Galilean invariance, which means that an internal observer moving with the hydrodynamic current and observing individual molecules would not be able to detect the current. In a metal the electrons disengage from their parent positive ions. The ions vibrate around fixed positions, not participating in the current. When electrical current j flows, the electrons have a net drift velocity which can be detected by a local observer watching ions as well as electrons. The system does not have Galilean invariance. Collisions of electrons do not conserve j , and act to restore electrons to the $j = 0$ state of thermal equilibrium, not the moving local equilibrium of hydrodynamics. This is the source of resistance in metals. Electrical current is less “collective” than hydrodynamic current. However, at very low temperature many metals become superconductors, seen in the data of Fig. 2. This is a different state of electronic matter in which electrons behave more collectively than they do in the “normal” or higher-temperature resistive state of metallic electronic matter. The collective (and quantum-mechanically phase-coherent) superconducting state allows $j \neq 0$ with $E = 0$, thus making $\sigma = \infty$. See SUPERCONDUCTIVITY.

Electrons in quantum theory are waves as well as particles, and can diffract around the crystalline

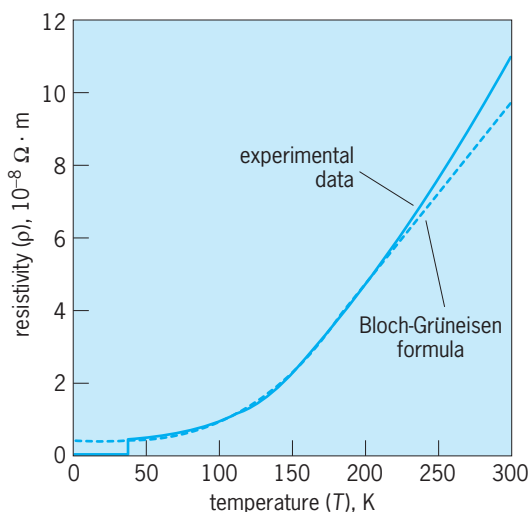


Fig. 2. Electrical conductivity of magnesium diboride (MgB_2), plotted, as is customary for metals, as resistivity (ρ) versus temperature (T). The unit $10^{-8} \Omega \cdot \text{m}$ is often written as 1 micro-ohm cm or $1 \mu\Omega\text{-cm}$. Below $T = 40 \text{ K} = -233^\circ\text{C}$, MgB_2 is a superconductor with $\rho = 0$. A fit using Bloch-Grüneisen theory, explained in the text, is also shown. (Data from Z. X. Ye et al., *Electron scattering dependence of dendritic magnetic instability in superconducting MgB_2 films*, *Appl. Phys. Lett.*, 86:5284–5286, 2004)

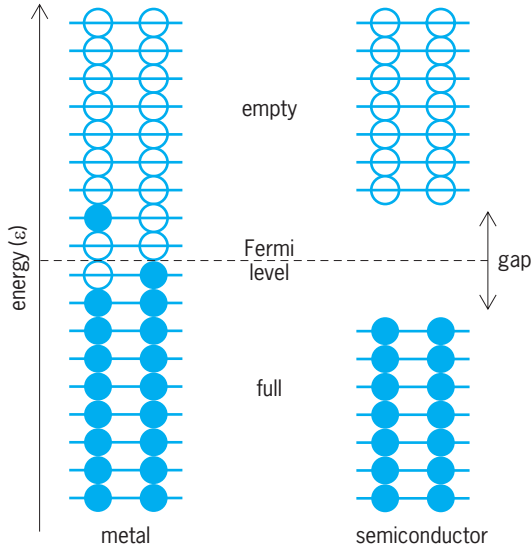


Fig. 3. Schematic energy spectrum in a metal versus a semiconductor. Filled and empty circles represent full and empty quantum states, two per level corresponding to the two spin orientations, just as in an atom. Adjacent levels are infinitesimally close except at the Fermi level of a semiconductor, which lies in the energy gap where there are no quantum states. The metal states are shown with a filled state above the Fermi level and a vacant state or hole below the Fermi level, as would happen by ordinary thermal excitation at room temperature. Semiconductors are likely to have few thermal excitations. In such a state the conductivity σ would be close to zero.

lattice of ions. In perfect crystals, if ions were not vibrating, the electron waves would remain coherent over the whole sample size, and conductivity would be infinite. The actual finite value occurs because electrons scatter from defects in the lattice, from atomic vibrations, and from each other. The last source of scattering is small, and the other sources are minimized in pure crystals at low temperature. Unlike molecules in water, which collide frequently, electrons in good metals collide infrequently and have long “mean free paths” or distances of travel between collisions. The collision frequency for water molecules at room temperature (300 K), denoted $1/\tau$, is about $6 \times 10^{12}/\text{s}$. This comes from dividing the thermal velocity, given by Eq. (2), by the distance $\sim 10^{-10}$ m between colli-

$$v_{\text{th}} = \sqrt{3k_B T/M} \approx 600 \text{ m/s} \quad (2)$$

sions. For electrons in metals the frequency of collisions with atomic vibrations at $T = 300$ K is given by Eq. (3). In these formulas, k_B is Boltzmann's

$$1/\tau \approx (1 - 10)k_B T/\hbar \sim 10^{14}/\text{s} \quad (3)$$

constant, M is the mass of a water molecule, and \hbar is Planck's constant divided by 2π . Compared to atoms, electrons have much higher mean velocities, $v_{\text{el}} \sim (2-10) \times 10^5$ m/s, and v_{el} is independent of temperature. This is a quantum effect caused by the Pauli principle. In a free-electron model, where all diffraction from atoms is ignored, the value is called the

Fermi velocity and is given Eq. (4), where n is the

$$v_F = \hbar(3\pi^2 n)^{1/3}/m \quad (4)$$

number of electrons per unit volume, of the order of 10^{29} m^{-3} , and m is the electron mass. As a consequence of the larger velocity, the mean free path for electrons, estimated as velocity divided by collision rate, is typically 10 to 20 interatomic spacings, 50 times longer than for water molecules. See CRYSTAL DEFECTS; EXCLUSION PRINCIPLE; FREE-ELECTRON THEORY OF METALS; LATTICE VIBRATIONS; QUANTUM THEORY OF MATTER.

Bloch-Boltzmann theory. The long mean free path of electrons in metals is the key to formulating a theory. In a perfect crystal the quantum description of electrons as particle-wave entities is similar to the idea of electromagnetic waves confined in a resonant cavity. The wavelengths have to “fit” the dimensions of the cavity, which causes quantization of wavelength λ . The individual quantum states or orbitals are labeled by their wavevectors $k = 2\pi/\lambda$. The orbital labeled by k has an energy $\varepsilon(k)$, and is pictured as a wave oscillating with a space and time dependence given approximately by $\cos [kx - \varepsilon(k)t/\hbar]$, like a traveling electromagnetic wave with frequency $\omega(k) = \varepsilon(k)/\hbar$. A wave packet built from such orbitals moves with velocity given by Eq. (5). There is no sim-

$$v(k) = [d\varepsilon(k)/dk]/\hbar \quad (5)$$

ple formula for the function $\varepsilon(k)$. It has to be measured by photoemission or computed using band theory, and varies from metal to metal. The wave packets in real metals do not remain phase-coherent over the whole sample, only over the distance between collisions. Provided this distance is longer than a wavelength, the wavelength and wavevector remain meaningful quantities, which can be used to build a theory, as was done by Felix Bloch in 1928.

Bloch's theory starts from the recognition of Bloch orbitals with wavevectors k . It then adopts from Boltzmann the notion of the distribution $F(k)$ which gives the probability that the state k is occupied. Because of the Pauli principle, this probability cannot exceed 2 (one for spin “up” and one for spin “down”). For states of low energy [$\varepsilon(k)$ far below the Fermi level ε_F], the occupancy is very close to 2, and for states of high energy [$\varepsilon(k)$ far above the Fermi level ε_F], the occupancy is very close to 0. In thermal equilibrium the occupancy $F(k)$ is equal to the Fermi-Dirac function, given by Eq. (6). The con-

$$f(k) = \frac{2}{\exp\left[\frac{\varepsilon(k) - \varepsilon_F}{k_B T}\right] + 1} \quad (6)$$

ductivity is calculated by summing the velocities of the occupied states, yielding Eq. (7).

$$\sigma = j/E = \frac{-e}{E \cdot \text{Vol}} \sum_k v(k)F(k) \quad (7)$$

When the field E is zero, the distribution $F(k)$ is $f(k)$ in Eq. (6), which has $j = 0$ because states with

positive and negative velocities $v(k)$ are equally populated. The effect of the field E is to cause the k -vectors of the occupied states to change according to Bloch's law, Eq. (8). This is the quantum version

$$\hbar dk/dt = -eE \quad (8)$$

of Newtonian acceleration by the electric force $-eE$, using de Broglie's momentum $\hbar k$ of a quantum wave. The acceleration is continuously resisted by scattering. Therefore the total shift of the typical k -vector is given by Eq. (9), where τ is the time between col-

$$\Delta k \approx -eE\tau/\hbar \quad (9)$$

lisions. Then, using $F(k) \approx f(k - \Delta k)$, we can write the answer for the conductivity as Eq. (10), where $(n/m)_{\text{eff}}$ is given by Eq. (11).

$$\sigma = \left(\frac{n}{m}\right)_{\text{eff}} e^2 \tau \quad (10)$$

$$\begin{aligned} \left(\frac{n}{m}\right)_{\text{eff}} &= \frac{-1}{\hbar \text{Vol}} \sum_k v(k) \frac{df(k)}{dk} \\ &= \frac{1}{\text{Vol}} \sum_k \left[\frac{d^2 \varepsilon}{\hbar^2 dk^2} \right] f(k) \end{aligned} \quad (11)$$

See NONRELATIVISTIC QUANTUM THEORY.

The last part of Eq. (11) is derived by integration by parts after substituting the expression given by Eq. (5) for $v(k)$, and can be interpreted as electron density, given by Eq. (12), times reciprocal effective mass, given by Eq. (13). The Bloch theory recovers a

$$n = \left(\sum f(k) \right) / \text{Vol} \quad (12)$$

$$1/m^* = d^2 \varepsilon / d(\hbar k)^2 \quad (13)$$

result proposed on classical grounds by Paul Drude shortly after the discovery of the electron, namely $\sigma = ne^2\tau/m$. However, the interpretation is quite different. A proper quantum theory for the scattering rate τ is needed, and was provided by Bloch by making a quantum generalization of the Boltzmann scattering operator. See BOLTZMANN TRANSPORT EQUATION.

An approximate solution of Bloch's theory, known as the Bloch-Grüneisen formula, is often very successful in fitting data, as shown in Fig. 2 for magnesium diboride (MgB_2). In this example, theory deviates from experiment at higher temperatures because high-frequency lattice vibrations, involving boron atoms, are not well represented by the form of the approximate theory. The extrapolated $T = 0$ resistivity value, $\rho_0 = 0.4 \times 10^{-8} \Omega \cdot \text{m}$, is called the residual resistivity, and would be zero in a perfect crystal.

Bloch's theory has failures, but they are outnumbered by the successes. One failure is that collective behavior can sometimes be seen at low temperature, the outstanding example being superconductivity. Other examples are more controversial, and include so-called Luttinger liquid behavior, which definitely happens in ideal one-dimensional theoretical models and probably is seen in careful experiments on

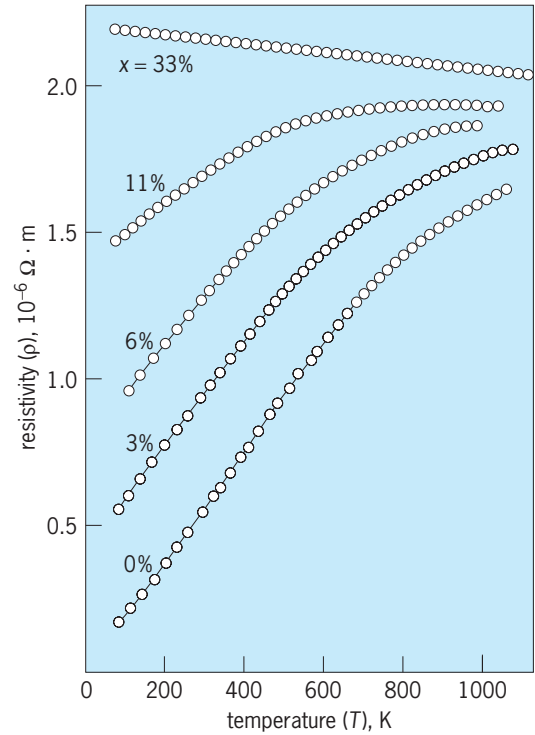


Fig. 4. Resistivity ρ of $\text{Ti}_{1-x}\text{Al}_x$ alloys as function of temperature (T) for various aluminum concentrations x . (From J. H. Mooij, *Electrical conduction in concentrated disordered transition-metal alloys*, *Phys. Stat. Sol. (a)*, 17: 521-530, 1973)

such systems as carbon nanotubes. Another case is transport by so-called sliding charge-density waves, perhaps seen in quasi-one-dimensional metals like NbSe_3 . A more common failure is that in metals which are very impure or which have very strong scattering, the mean free path between scattering events may become so short that the wavelength and wavevector of electron quantum states is no longer definable. This undermines the basis of Bloch's theory. See CHARGE-DENSITY WAVE.

Such a failure of the theory (technically, a failure of the quasiparticle picture) is illustrated in Fig. 4, showing the resistivity of $\text{Ti}_{1-x}\text{Al}_x$ alloys. At the lower aluminum concentrations, $x = 3\%$ and 6% , the $\rho(T)$ curves shift upward, obeying Matthiessen's rule, given by Eq. (14), where the residual resistivity

$$\rho = \rho_0 + \rho_{\text{pure}}(T) \quad (14)$$

ρ_0 is proportional to impurity concentration x . For the larger concentrations, a too short mean free path causes the quasiparticle picture underlying Bloch-Boltzmann theory to fail. This in turn causes failure of Matthiessen's rule. The quasiparticle picture also fails at high temperature for all concentrations x . See MATTHIESSEN'S RULE.

A more abstract formulation of nonequilibrium effects like conductivity exists under the name of Kubo formulas. These provide a route to improved theories.

Philip B. Allen

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Electrical connector

A device that joins electrical conductors mechanically and electrically to other conductors and to the terminals of apparatus and equipment. The term covers a wide range of devices designed, for example, to connect small conductors employed in communication circuits, or at the other extreme, large cables and bus-bars.

Electrical connectors are applied to conductors in a variety of ways. Soldered connectors have a tube or hole of approximately the same diameter as the conductor. The conductor and connector are heated, the conductor inserted, and solder flowed into the joint until it is filled. Solderless connectors are applied by clamping the conductor or conductors in a bolted assembly or by staking or crimping under great mechanical force, usually by means of special tools designed for the purpose.

Industrial and power types. Connectors for electric power systems are commonly cast and assembled with bolts. The material most commonly used is copper alloy, which may vary in composition and properties depending upon the intended use of the connector. Where heavy currents flow through the body of the connector, an alloy of relatively high conductivity is used. Where the connector serves principally to clamp conductors together, an alloy of higher strength and lower conductivity may be used. An important consideration governing the choice of alloy is that it must have a coefficient of thermal expansion very close to that of the conductor material itself (copper or aluminum), so that the connection will remain secure through wide temperature changes.

Connectors designed for severe outdoor service and heavy current and mechanical loading are called power connectors and are widely used in power substations for cable-to-stud connections at equipment, cable-to-bus, bus-to-bus, and bus-to-line connections.

For industrial application, more compact constructions and greater versatility may be required, and connectors with such features are called industrial connectors. In practice, both classes are employed in power systems and industrial plants, the choice depending upon the actual service conditions.

Typical connector types are in-line splice couplers, T-tap connectors, terminal lugs, and stud connectors. Couplers join conductors end to end. T-tap connectors join a through conductor to another conductor at right angles to it (Fig. 1a). Terminal lugs join the conductor to a drilled tongue for bolting to the terminals of equipment (Fig. 1b). Stud connectors join

the conductor to equipment studs; the stud clamp is threaded or smooth to match the stud. Many variations of these types are made.

Split-bolt connectors are a compact construction widely used for splices and taps in building wiring. The bolt-shape casting has a wide and deep slot lengthwise. The conductors are inserted in the slot and the nut is drawn up, clamping the conductors together inside the bolt (Fig. 1c). Newer types of connectors are made to serve this same function.

Expansion connectors or flexible connectors allow some limited motion between the connected conductors. The clamp portions of the connector are joined by short lengths of flexible copper braid and may also be held in alignment by a telescoping guide.

Heavily tinned copper-alloy bolted connectors or aluminum-alloy-body connectors may be used to connect aluminum conductors. They are applied with an oxide-inhibiting compound. Where copper and aluminum conductors are to be clamped together, they are separated by bimetal or tinned washers to prevent galvanic corrosion.

In electronic equipment assembly, involving hookup of wires to terminals and of other components onto printed circuit boards, connections are made by soldering the wire or component to the terminal or printed circuit board. Although soldered terminals are by far the most common type of wiring connections in electronic chassis applications and in manufacture of electronic and electrical equipment, the pressure-wrap connection is a tool-applied method used in the field that speeds up connections where very many have to be made, as in wiring telephone and signal and control equipment. The tool that applies the wire wrap to the terminal stud assures an extremely tight, low-resistance, high-reliability connection by slightly elongating the wire as it is wrapped, putting a tight constrictive force on the terminal.

Separable types. These consist of matched plugs and receptacles, which may be readily separated to disconnect a conductor or group of conductors from the circuit or system. Separable connectors are commonly used for the connection of portable appliances and equipment to an electric wiring system. In electronics and communication they are used for connecting various components so that a single component can be readily removed for servicing without disturbing the circuitry.

For connections to an electric power supply, separable connectors consist of receptacles, usually fixed and permanently connected to the wiring system,

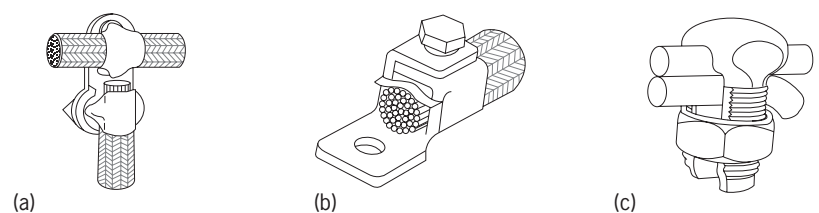


Fig. 1. Types of connectors. (a) T-tap connector. (b) Terminal lug. (c) Split-bolt connector.